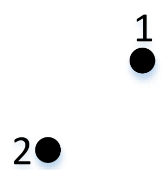
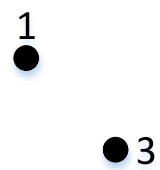
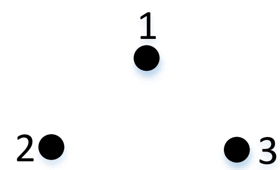
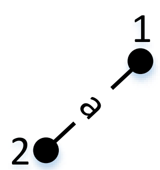
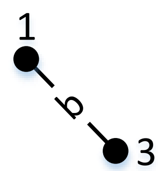
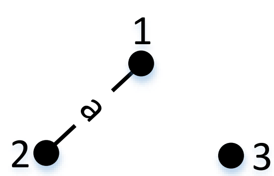
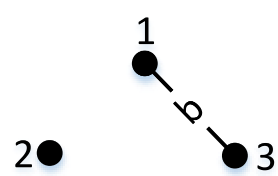
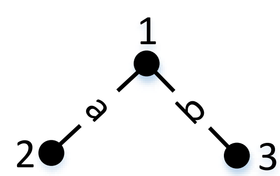
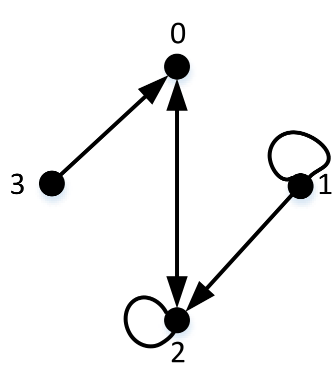
PART A – GRAPH THEORY

1. <u>Subgraphs and Connectivity (10 marks)</u>

null graph Connected: Y	 Connected: Y	 Connected: Y	 Connected: Y
 Connected: N	 Connected: N	 Connected: N	 Connected: N
 Connected: Y	 Connected: Y		
 Connected: N	 Connected: N		 Connected: Y

2. <u>Matrices in Graph Theory (10 marks)</u>

a)



b)

i \ j	0	1	2	3
0	1	0	1	0
1	1	1	2	0
2	1	0	2	0
3	0	0	1	0

PART B – SEQUENCES, RECURRENCE RELATIONS – 10 MARKS

1. <u>Terms of the Sequence (4 marks)</u>

$a_1 = 5 + 1 + 2^1 = 8$
$a_2 = 5 + 1 + 2^1 + 2 + 2^2 = 14$
$a_3 = 5 + 1 + 2^1 + 2 + 2^2 + 3 + 2^3 = 25$
$a_4 = 5 + 1 + 2^1 + 2 + 2^2 + 3 + 2^3 + 4 + 2^4 = 45$

2. <u>Iteration (6 marks)</u>

$$a_n = 5 + \sum_{i=1}^{n} i + \sum_{i=1}^{n} 2^i = 5 + \frac{n(n+1)}{2} + \sum_{i=0}^{n} 2^i - 2^0$$
$$= 5 + \frac{n(n+1)}{2} + 2^{n+1} - 1 - 1 = 2^{n+1} + \frac{n(n+1)}{2} + 3$$

PART C – INDUCTION – 20 MARKS

1. Set D (1 mark)

$\mathbb{N}^+$

2. P(n) (4 marks)

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

3. Basic Step of the Proof (4 marks)

For n=1

$$\sum_{i=1}^{n} i^2 = \sum_{i=1}^{1} i^2 = 1^2 = 1$$

$$\frac{n(n+1)(2n+1)}{6} = \frac{1(2)(3)}{6} = \frac{6}{6} = 1$$

So P(1) is true.

4. Inductive Step of the Proof (11 marks)

Assume that P(k) is true for some k≥1, i.e. that $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$ (Inductive Hypothesis)

We will show that P(k+1) is true, i.e. that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$

$$\sum_{i=1}^{k+1} i^2 = (k+1)^2 + \sum_{i=1}^{k} i^2$$

$$= (k+1)^2 + \frac{k(k+1)(2k+1)}{6}$$ by Inductive Hypothesis

$$= \frac{6(k+1)^2 + k(k+1)(2k+1)}{6} = \frac{(k+1)(6(k+1) + k(2k+1))}{6} = \frac{(k+1)(2k^2+7k+6)}{6}$$

We want to show that this equal to $\frac{(k+1)(k+2)(2k+3)}{6}$ i.e. that $2k^2 + 7k + 6 = (k+2)(2k+3)$

$(k+2)(2k+3) = 2k^2 + 3k + 4k + 6 = 2k^2 + 7k + 6$

QED